

UK Junior Mathematical Olympiad 2009

Organised by The United Kingdom Mathematics Trust

Tuesday 16th June 2009

RULES AND GUIDELINES : READ THESE INSTRUCTIONS CAREFULLY BEFORE STARTING

1. Time allowed: 2 hours.
2. **The use of calculators, measuring instruments and squared paper is forbidden.**
3. All candidates must be in *School Year 8 or below* (England and Wales), *S2 or below* (Scotland), *School Year 9 or below* (Northern Ireland).
4. For questions in Section A *only the answer is required*. Enter each answer neatly in the relevant box on the Front Sheet. Do not hand in rough work. Write in blue or black pen or pencil.
For questions in Section B you must give *full written solutions*, including clear mathematical explanations as to why your method is correct.
Solutions must be written neatly on A4 paper, starting each question on a fresh sheet.
Sheets must be STAPLED together in the top left corner with the Front Sheet on top.
Do not hand in rough work.
5. Questions A1-A10 are relatively short questions. Try to complete Section A within the first 45 minutes so as to allow well over an hour for Section B.
6. Questions B1-B6 are longer questions requiring *full written solutions*.
This means that each answer must be accompanied by clear explanations and proofs.
Work in rough first, then set out your final solution with clear explanations of each step.
7. These problems are meant to be challenging! Do not hurry. Try the earlier questions in each section first (they tend to be easier). Try to finish whole questions even if you can't do many. A good candidate will have done most of Section A and given solutions to at least two questions in Section B.
8. Answers must be FULLY SIMPLIFIED, and EXACT using symbols like π , fractions, or square roots if appropriate, but NOT decimal approximations.

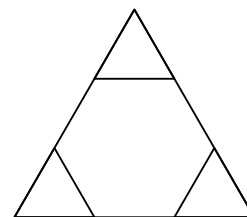
DO NOT OPEN THE PAPER UNTIL INSTRUCTED BY THE INVIGILATOR TO DO SO!

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Section A

A1 What is the value of $200^2 + 9^2$?

A2 The diagram shows a regular hexagon inside an equilateral triangle. The area of the larger triangle is 60 cm^2 . What is the area of the hexagon?



A3 The positive whole numbers a , b and c are all different and $a^2 + b^2 + c^2 = 121$. What is the value of $a + b + c$?

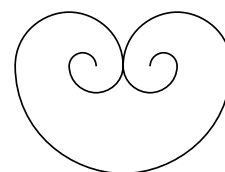
A4 The sum of three numbers is 2009. The sum of the first two numbers is 1004 and the sum of the last two is 1005. What is the product of all three numbers?

A5 Andrea's petrol tank holds up to 44 litres of fuel. She goes to the garage when her tank is a quarter full and puts more petrol in the tank until it is two-thirds full. How many litres of petrol does she put in the tank?

A6 The shorter sides of a right-angled isosceles triangle are each 10 cm long. The triangle is folded in half along its line of symmetry to form a smaller triangle. How much longer is the perimeter of the larger triangle than that of the smaller?

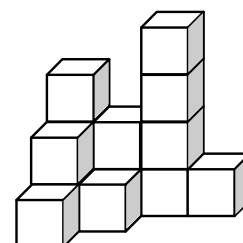
A7 Dean runs on a treadmill for thirty minutes. To keep his mind active as well as his legs, he works out what fraction of the total time has passed at each half minute and minute from the start. How many of the results of his calculations can be expressed in the form $\frac{1}{n}$, where n is an integer greater than 1?

A8 The diagram shows a curve made from seven semicircular arcs, the radius of each of which is 1 cm, 2 cm, 4 cm or 8 cm. What is the length of the curve?



A9 A book has 89 pages, but the page numbers are printed incorrectly. Every third page number has been omitted, so that the pages are numbered 1, 2, 4, 5, 7, 8, ... and so on. What is the number on the last printed page?

A10 Gill piles up fourteen bricks into the shape shown in the diagram. Each brick is a cube of side 10 cm and, from the second layer upwards, sits exactly on top of the brick below. Including the base, what is the surface area of Gill's construction?



Section B

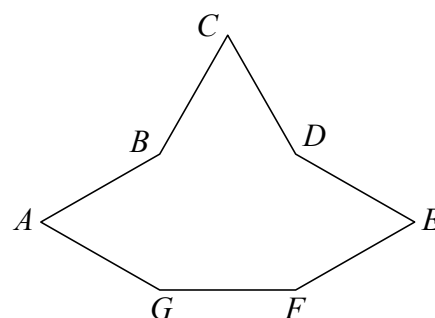
Your solutions to Section B will have a major effect on your JMO results. Concentrate on one or two questions first and then **write out full solutions** (not just brief 'answers').

B1 In 2007 Alphonse grew twice the number of grapes that Pierre did. In 2008 Pierre grew twice the number of grapes that Alphonse did. Over the two years Alphonse grew 49 000 grapes, which was 7600 less than Pierre. How many grapes did Alphonse grow in 2007?

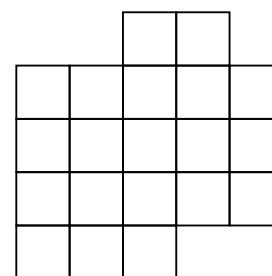
B2 $ABCD$ is a square. The point E is outside the square so that CDE is an equilateral triangle. Find angle BED .

B3 Tom left a motorway service station and travelled towards Glasgow at a steady speed of 60 mph. Tim left the same service station 10 minutes after Tom and travelled in the same direction at a steady speed, overtaking Tom after a further 1 hour 40 minutes. At what speed did Tim travel?

B4 The diagram shows a polygon $ABCDEFG$, in which $FG = 6$ and $GA = AB = BC = CD = DE = EF$. Also $BDFG$ is a square. The area of the whole polygon is exactly twice the area of $BDFG$. Find the length of the perimeter of the polygon.



B5 An ant wishes to make a circuit of the board shown, visiting each square exactly once and returning to the starting square. At each step the ant moves to an adjacent square across an edge. Two circuits are considered to be the same if the first follows the same path as the second but either starts at a different square or follows the same path in reverse. How many such circuits are possible?



B6 I want to choose a list of n different numbers from the first 20 positive integers so that no two of my numbers differ by 5. What is the largest value of n for which this is possible? How many different lists are there with this many numbers?